Functions

Create a Roller Coaster Ride

Grade 8

45 Min.

or

LESSON OBJECTIVE Students will use nonlinear functions to model a roller coaster ride

LANGUAGE OBJECTIVES Using mathematical language and linking words and phrases to connect ideas, students will describe the use of nonlinear functions in modeling real-world situations.

PREREQUISITE SKILLS Students graph functions and describe translations in the coordinate plane.

Teachers can use the Imagine Math Standards Report and the Benchmark Performance Level Report to evaluate student readiness to complete this task.

COLLEGE AND CAREER READINESS STANDARDS FOR MATHEMATICS 8.F.B.5

CCSS MATH 8.F.B.5	**TEKS MATH** 8.5.H	**QSC** 209

Teacher Preparation

LESSON OVERVIEW Students develop plans for sections of a roller coaster in the coordinate plane using sections of the graphs of different nonlinear functions. They write equations for the translations of the graphs and tell where the height of the roller coaster is increasing or decreasing. They solve equations to check their answers.

MATERIALS
- Graphing Paper
- Drawing Paper
- Vocabulary Knowledge Rating Sheet

Understand Science Background

The concept of roller coasters can be traced back to the Russian "ice hills" of the 1400s, when people paid to climb stairs to the tops of hills built from snow and ice to ride down the hills in sleds. In the 1700s, wheels were added to the sleds, so people could ride the ice hills year-round. The craze for these rides, known as "Russian Mountains," spread throughout Europe until the mid-1800s, when increasing injuries caused most to be shut down.

The roller coaster concept reappeared in the United States in 1874, when an inclined railway at a Pennsylvania coal mine was repurposed as an amusement ride. Mules pulled railway cars up the hill, and gravity brought cars and riders, including mules, back down. The basic principle of all roller coasters, from the early ice hills to the high-tech roller coaster rides at today's theme parks, is that an initial height must be reached to build momentum, and then gravity does the rest.

As part of the roller coaster ride, there are smaller hills and valleys, as well as twists, turns, and even loops. Because energy is lost due to friction, hill heights must keep decreasing so that the car keeps moving until it reaches the bottom.

Collaborate: Work with science, literacy, and history teachers to explore opportunities to expand cross-curricular experiences for students.

Building a modern-era roller coaster is a major design and engineering project. Initial design is done with scale models on computers, using mathematical models such as quadratic functions to simulate the ride. Benefits of creating scale models include the ability to create new and exciting rides for all ages while ensuring that the roller coaster will work safely for passengers based on principles of physics.

After computer models and prototypes are completed, the entire ride is manufactured and installed on-site. Before a ride is open to the public, it undergoes extensive testing. This testing is continued even after it is operational, with daily inspections. A major goal of modern-era roller coaster designers and engineers is to keep the rides safe, avoiding the fate of the "Russian Mountains."

Pre-Task Class Activities

Engage in Discussion

Engage students in a discussion using their prior knowledge of and personal experiences with nonlinear functions. Guide the discussion toward how creating models can help manufacturers or customers conceptualize what they will produce or purchase to help students understand the **Essential Question**. Direct students to use the **Supporting Words** and **Linking Words and Phrases** on student pages 2 and 3 during their conversations. You may also use sentence stems to support your students' language usage in context.

Possible Discussion Topics

1. Have students describe a roller coaster ride they enjoy and why they enjoy it.
   - A ride I enjoy is ______ because it has ______.
2. Discuss what makes a roller coaster fun and exciting, including hills and valleys, twists and turns, loops, tunnels, and other features.
   - As a result, this feature ______.
3. Use the **Background** to explain why the hills in a roller coaster must keep decreasing in height.
   - Due to ______, roller coaster hill heights must ______.
4. Find, share, and discuss examples of objects that can be modeled with nonlinear functions.
   - To begin with, a nonlinear function is ______.
   - To illustrate, you could model ______ with a nonlinear function.

Examples of Situations Where Nonlinear Functions Can Be Used as a Model
• Roller coaster design • Path of a tossed ball • Product designs such as chairs or tables • Plans for houses, malls, or parks

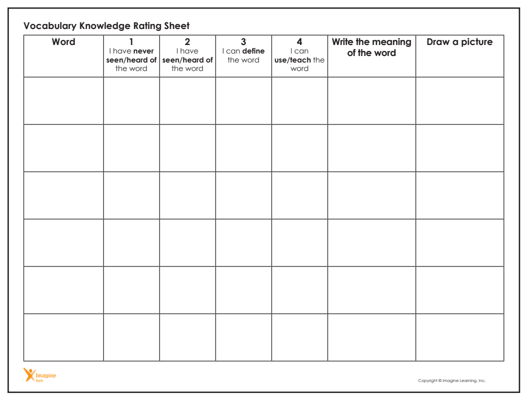
5. Discuss how using nonlinear functions to create scale models can help designers, engineers, and manufacturers conceptualize what they will produce.
   - Because designers, engineers, and manufacturers ______, they can ______.
   - With this in mind, creating a scale model could help by ______.

Review Vocabulary

Based on your students' language needs, use these activities to provide additional vocabulary support. Students should be prepared to meet the **Language Objective**.

Vocabulary Knowledge Rating Sheet

Ask students to choose four words from the **Supporting Words** and **Major Words** to write on a Vocabulary Knowledge Rating Sheet. Ask students to self-assess their knowledge of the words using the ratings 1–4 as shown on the sheet. Ask students to write what they think the word means in their own words. Then have them draw a picture of the word on their rating sheet.


Vocabulary Knowledge Rating Sheet

Word	1 I have **never seen/heard of** the word	2 I have **seen/heard of** the word	3 I can **define** the word	4 I can **use/teach** the word	Write the meaning of the word	Draw a picture

Cognates

Cognates are words from two different languages that share a common language origin. As a result, cognates often look or sound similar. The table shows some English and Spanish cognates from this lesson's vocabulary words. Cognates may similarly exist for other languages.

English	Spanish
continuous	*continua*
friction	*fricción*
kinetic energy	*energía cinética*
linear	*lineal*
nonlinear	*no lineal*
potential energy	*energía potencial*
translation	*translación*

Additional Vocabulary Words

Every class has its own language needs. Based on your students' proficiency, you may wish to review some of the **Additional Words** used in this lesson. It may also be helpful to allow students to look up definitions of the **Supporting Words** and **Major Words** in their native languages.

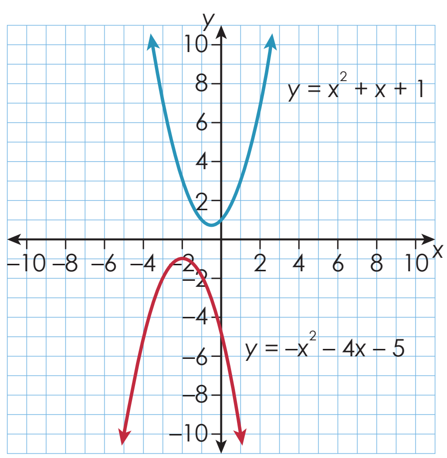
ADDITIONAL WORDS

equation: a mathematical sentence with an equal sign that shows that two expressions have equal value

function: A relationship between a set of inputs and corresponding outputs where there is exactly one output for each input. Functions can be represented as a set of ordered pairs, as an equation, and as a graph. Function equations are usually written in the form: $f(x) = x + 3$ or $y = x + 3$.

graph: a pictorial representation of a mathematical relationship

parabola: A set of points that are equidistant from a point. A graph of a quadratic function in the form $y = ax^2 + bx + c$.



Review Math Concepts

Depending on the needs of the students, use these teaching strategies to review and verify students' ability to graph functions and describe translations in the coordinate plane. Throughout the math review, maintain focus on the **Essential Question**.

Jigsaw

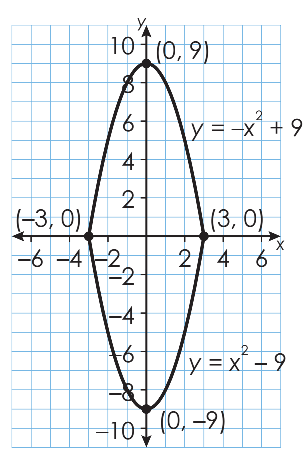
Provide graph and drawing paper to each group of 4 to 6 students. Students will work together as a group to create a product line of furniture, clothing, or other items, using designs based on translations of the functions $y = x^2$ and $y = -x^2$ for $-3 \leq x \leq 3$. Groups should produce graphs as well as sketches of their finished products. (Sample response: The graph of $y = x^2$ is translated 9 units down, and the graph of $y = -x^2$ is translated 9 units up. The design is repeated horizontally on wooden chairs and tables.) Under the graphs and sketches each group should describe their translations and products, using cloze sentences as necessary.

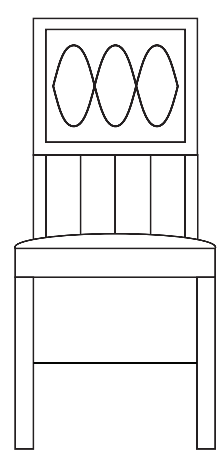
- The graph of $y = x^2$ is translated ______.
- The graph of $y = -x^2$ is translated ______.
- This design will be used ______.

Group members should work together to define the product line, create designs, describe translations, and sketch products. Upon completion, create new groups, mixing the original groups as much as possible with others. Ask each group member to present their product lines, including graphs and sketches, and respond to questions from their new group members.

Translations

Using one of the student-created designs from the **Jigsaw** activity, guide the whole class through translating sections of the graphs of $y = x^2$ and $y = -x^2$. Solicit student interaction to draw and label the translated graphs and to write their equations. Your completed diagram should resemble those on the first page and in **section A**. Allow this model to remain on display throughout the task for student reference. Translated graphs and a chair design for the **Jigsaw** sample response are presented here.





The Application Task

Application Tasks are Performance Tasks where students apply their conceptual understanding and use procedural skills to solve a real-world problem. Application Tasks provide students the opportunity to demonstrate understanding and proficiency in multiple ways. Use the following ideas and modifications to ensure that students clearly understand the purpose, context, and constraints of the task. As students become more familiar with Application Tasks, less guidance may be needed.

Introduce the Essential Question

Direct students to the **Essential Question** on page 1. In this task, students will model real-world situations with nonlinear functions.

Make sure that students understand that the real-world situations in the **Essential Question** refer to the **Goal** of using nonlinear functions to model a roller coaster ride.

Analyze Instructions and Background

You may wish to provide support for students by reading and analyzing the instructions and background.

First, ask students to individually read the instructions beginning with "In this task, you are . . ." ***for context only***. Students will share with a partner their answer to the questions "What is the situation?" and "What do we need to find?" Note that these questions clarify the **Essential Question**: How can you model real-world situations with nonlinear functions? Provide the sentence stems below for additional language support. Circulate the room and assess student responses, providing appropriate feedback.

- This activity has to do with ______.
- I need to model ______.

Continue the class discussion as you read the remainder of page 1 and the **Background** information on page 2. Ask students to read the pages either independently or in groups, and then answer the **Think about It** question. After reviewing these pages, students should be able to understand the **Goal** of using nonlinear functions to model a roller coaster ride.

Now, ask students to read or scan the first two pages again, this time ***for mathematical content***. Ask students to work with a partner to answer the questions "What numbers appear in the problem?" and "What do those numbers represent (including the units)?" Ask students to share their answers aloud. Provide appropriate feedback.

- Some numbers I will work with are ______, ______, and ______.
- These numbers represent ______ and are measured in ______, which means ______.

Then ask students to pay close attention to the constraints: "What limitations are given in the problem?" Students should also be able to answer, "How does the **Sample Plan** meet the conditions set by the constraints?"

- One constraint is that ______ must be ______.
- The **Sample Plan** meets the constraints because ______.

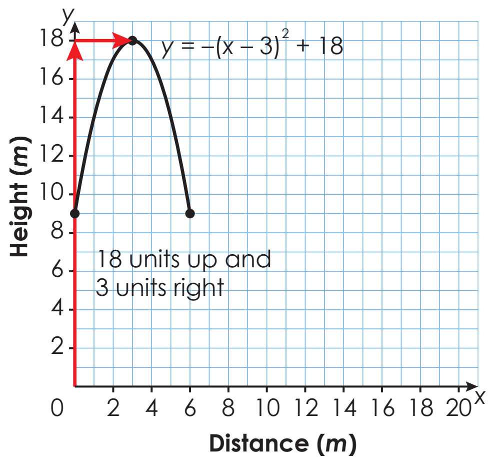
Walk Through the Sample Plan

The **Sample Plan** on page 1 provides students with a worked solution to the task. While reviewing the **Sample Plan** with your class, elicit participation as much as possible. You may also need to clarify the information in the images and tables on pages 1 and 2.

The **Essential Question** asks us to use nonlinear functions to model a real-world situation. Specifically, we need to create a roller coaster ride using 4 nonlinear graph sections. Consequently, the **Sample Plan** begins by selecting graph sections from the 6 options given on page 1. One constraint in the task requires that there must be at least 2 hills in each ride, so 2 graph sections will be chosen from the $y = -x^2$ graphs, which have the shape of a hill. Then, because we are trying to create the shape of a roller coaster, the other 2 graph sections should be chosen from the $y = x^2$ graphs, which have the shape of a valley.

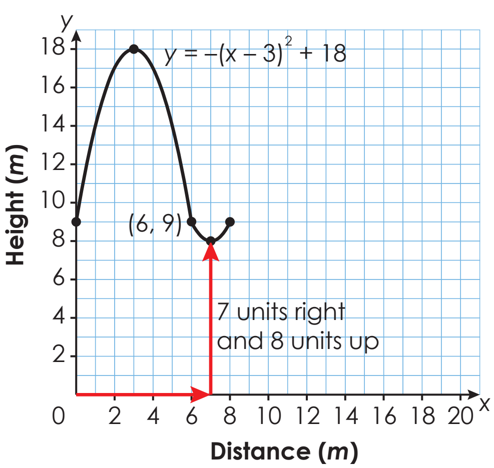
The first graph section in the **Sample Plan** is from the graph of $y = -x^2$, for the interval $-3 \le x \le 3$.

We can now translate this graph section into its position in the roller coaster ride. Since another constraint requires that the heights of hills must decrease from left to right, we need to shift this first graph section, or hill, up high enough so that the maximum height of the next hill will be lower. Since we are creating the model in a coordinate plane with a vertical axis that ends at $y = 18$, shifting the graph 18 units up makes the first hill as high as possible. Then, since the ride starts at $x = 0$, we also shift the graph section 3 units to the right, so the starting point is aligned with the y-axis. These two translations allow us to meet the remaining constraint that graph sections are shifted up and to the right in whole-number increments.



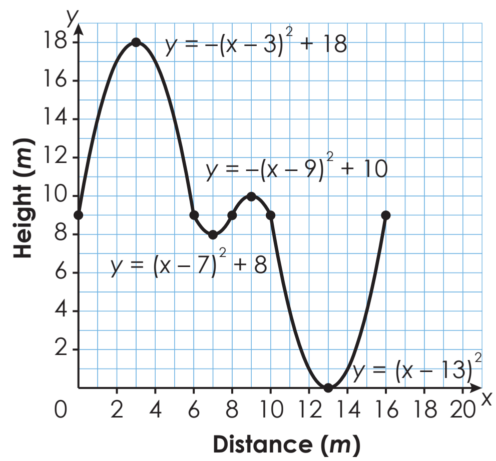
To write the equation for the translated graph section, we can use the equation $y = -(x - h)^2 + k$ in **section B**, where h is units shifted right and k is units shifted up. Substituting 3 for h and 18 for k gives the equation $y = -(x - 3)^2 + 18$.

Next, we append the second graph section to the roller coaster, which is from the graph of $y = x^2$, for the interval $-1 \le x \le 1$. This section must be continuous with the previous section (see the **Watch Out: Common Misconceptions** section for an explanation). Because the second graph section starts at (–1, 1), we shift it 7 units right and 8 units up, moving the starting point to (6, 9), which is where the previous graph section ends.



To write the equation for this translated graph section, we use the equation $y = (x - h)^2 + k$ on page 4, where h is units shifted right and k is units shifted up. Substituting 7 for h and 8 for k results in the equation $y = (x - 7)^2 + 8$.

The remaining graph sections are from $y = -x^2$, for the interval $-1 \le x \le 1$ (the second hill), and $y = x^2$, for the interval $-3 \le x \le 3$ (the second valley). The ending point of the second roller coaster graph section is at (8, 9), while the third graph section starts at (–1, –1) before translation. To keep the roller coaster continuous, we shift the third graph section 9 units right and 10 units up so its new starting point is at (8, 9). Similarly, the ending point of the third graph section is now at (10, 9), while the fourth graph section starts at (–3, 9) before translation, so we shift the fourth graph section 13 units right to put its new starting point at (10, 9).



Using the pattern detailed in **section B** to write the equations gives the equations $y = -(x - 9)^2 + 10$ and $y = (x - 13)^2$, respectively.

Our last step is to identify intervals where the roller coaster heights are increasing and decreasing. Note that these intervals are not the same as the intervals where different graph sections begin and end (see the **Watch Out: Common Misconceptions** section for further explanation). Moving from left to right, we see where vertical direction changes from up to down, and vice versa. The intervals are $0 < x < 3$, $7 < x < 9$, and $13 < x < 16$ for increasing heights, and $3 < x < 7$ and $9 < x < 13$ for decreasing heights.

Notice the relationships between the intervals and the equations, and how they describe the ups and downs of this roller coaster ride, addressing the **Essential Question** of how nonlinear equations can be used to model real-world situations. This section of our task is complete!

Introduce Section A

Help students understand that the purpose of **section A** is to analyze a roller coaster ride that has been created in the coordinate plane using graph sections of the nonlinear functions $y = x^2$ and $y = -x^2$. Students label each section of the roller coaster with the equation for the translation of the graph and describe where the roller coaster's height is increasing and decreasing.

Modifications: Students who need help labeling the roller coaster sections may benefit from using tracing paper to match the sections with the graphs on page 1 and counting how many units the graph has been shifted up and to the right on their own graph paper. If you are not using this task as an individual assessment, you could allow students to work in pairs on **section A**. Also consider discussing student responses as a class or in small targeted groups before continuing to **section B**. The **Talk about It** prompt on page 3, which asks about alternative problem-solving strategies, could lead to a productive and informative class discussion.

Introduce Section B

Help students understand that the purpose of **section B** is to create their own roller coaster rides using graph sections from page 1.

Modifications: Students who have difficulty completing **section B** may benefit from a review of the **Sample Plan** and the sample solution in **section A**. If you are not using this task as an individual assessment, consider having students draw their graphs on separate sheets of paper that you can later post around the room or in the hallway. You could also have a few students present their graphs while the remainder of the class asks questions and gives constructive criticism.

Introduce Section C

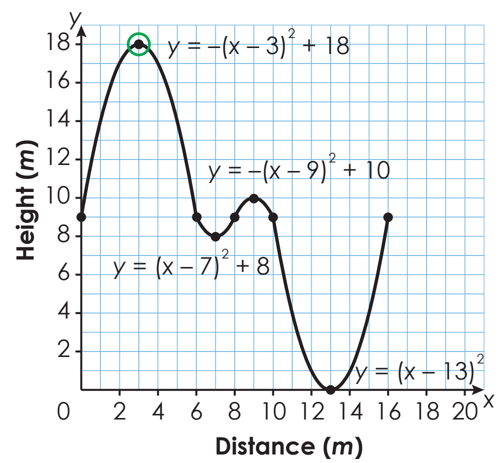
Help students understand that the purpose of **section C** is to describe where the height is increasing or decreasing in each of the roller coaster rides they create. They then connect their work to the overarching concept addressed in the **Essential Question** by explaining why the inequalities used describe where the height of the roller coaster is increasing or decreasing over time.

Students should use the **Linking Words and Phrases** as well as **Supporting Words** and **Major Words** in their responses.

Modifications: None.

Introduce Section D

Help students understand that the purpose of **section D** is to check the equations they wrote in **section B**, as well as their descriptions in **section C**, by substituting the x-value of the vertex to solve for y to check that the values match for each of the graph sections. Students then make sense of their work by explaining that the result of solving each equation for y should be the height of the roller coaster at the corresponding value of x for each vertex.



Equation	Vertex (x-value)	Solve for y
$y = -(x-3)^2 + 18$	3	$y = -(3-3)^2 + 18 = 18$

Modifications: Reinforce visual understanding by demonstrating the function input-output relationship using a graph. Connect the graph with the equation, and allow students to discuss similarities and differences between the graph representation and the equation representation. Encourage students to create a table to further show the connections between the multiple representations.

Watch Out: Common Misconceptions

Students may not understand why the translations of the graphs are represented by different equations. Demonstrate how this works by referring to **section A**. In this example, the first section of the roller coaster is a part of the graph of $y = -x^2$ that starts at the point (–2, –4). After the translation, this graph section starts at (0, 11), and the new equation is $y = -(x - 2)^2 + 15$. Substitute 0 for x, and solve for y:

$$\begin{aligned} y &= -(x - 2)^2 + 15 \\ &= -(0 - 2)^2 + 15 \\ &= -(-2)^2 + 15 \\ &= -(4) + 15 \\ &= -4 + 15 \\ &= 11 \end{aligned}$$

The value of y is the height at which the roller coaster starts, which is $y = 11$.

When describing intervals of the roller coaster ride where the height is increasing and decreasing, students may notice that these intervals are often constructed from parts of two of the different graph sections that are found on student page 1. Explain to students that the height of the ride is increasing when the value of y is increasing as x increases, and the height is decreasing when the value of y is decreasing as x increases.

Students may not understand what is meant by a continuous roller coaster ride. Refer them to the definition of continuous on student page 4: going on without a gap or interruption. Explain that this means there are no gaps or interruptions between sections of the graphs. Encourage students to visualize sections of the graphs as sections of the track in a real-life roller coaster. Gaps between sections of tracks in a real-life roller coaster could cause a serious accident!

Sample Rubric

Score	Criteria
3	Students accurately label the sections of the roller coaster ride and complete the tables in **section A**. In **section B**, they create two different roller coaster rides and accurately label each graph section. Their answers in **sections C** and **D** accurately apply math concepts. Students use linking words and phrases in their explanations, along with mathematical language.
2	Students label the sections of the roller coaster ride and complete the tables in **section A** with one error at most, and in **section B** they create two different roller coaster rides and label each graph section with minor errors. Their answers in **sections C** and **D** apply math concepts with no more than one or two minor errors. Students use linking words and phrases in their explanations, along with mathematical language.
1	Students label the sections of the roller coaster ride and complete the tables in **section A** with at least 50% accuracy, and in **section B**, they create two different roller coaster rides and label each graph section with at least 50% accuracy. Their answers in **sections C** and **D** are mostly inaccurate statements and demonstrate a limited understanding of math concepts. Students use mathematical language in their explanations, but they do not use linking words and phrases.
0	Students present a very limited or rudimentary response to the assignment. They label the sections of the roller coaster ride and complete the tables in **section A** with less than 50% accuracy, create two different roller coaster rides and label each of the graph sections in **section B** with less than 50% accuracy, and include very general or incomplete answers in **sections C** and **D**. Students do not use linking words and phrases in their explanations, and they do not use mathematical language.

STEM Activity Suggestions

STEM OBJECTIVE Analyze changes in potential and kinetic energy of a roller coaster.

NGSS SCIENCE	SCIENCE AND ENGINEERING PRACTICE	MATHEMATICAL PRACTICE
NGSS MS-PS3-5	SEP7	MP7

1. **Build Background** Roller coasters are amusement park rides that rely on the force of gravity. The cars on roller coaster rides do not have engines. Instead, after being mechanically pulled to the maximum height of the first hill, they travel along the tracks, gaining and losing energy as they go up and down hills, until their energy is diminished due to friction and the ride ends.
2. **Introduce the STEM Activity** In this activity, students will work in teams to combine, modify, and extend the roller coaster rides they created in the Application Task to design a longer roller coaster ride with many rises and falls. Students will analyze the relationship between potential energy and kinetic energy as a roller coaster car travels along their track. For example, at the start of a roller coaster ride, the car is pulled to the maximum height of the hill by an electric winch. At the top of that first hill, the car has maximum potential energy. From that point forward, the car loses kinetic energy as it is converted to potential energy when going uphill, and it loses potential energy as it is converted to kinetic energy when going downhill. Teams will present their rides as two-dimensional drawings in the coordinate plane. They will determine appropriate measurement units and scale for the height and distance of the roller coaster. They will also develop a visual strategy to explain the relationship between potential and kinetic energy as the car travels along the track. Teams will present their roller coaster rides to the class and should be prepared to offer reasonable arguments that their designs will work as planned, along with justifying their solutions.
3. **Guide Teams** Assist students as they create the designs for their roller coaster rides. Provide suggestions for how an understanding of the relationship between potential and kinetic energy can be applied:
   - Remind students that the roller coaster car has maximum potential energy when it is at the maximum height of the first hill after being pulled up mechanically. Point out that because of friction, the potential energy decreases as the ride continues. Guide students to understand that for this reason, the heights of the hills must decrease as the ride continues.
   - Students may want to add loops to their roller coasters. Point out that the height of a loop should be less than the hills or other loops that come before it. Encourage students to investigate centripetal force, which acts on an object moving in a circular path and which explains why loops shaped like tear drops are more common than circular loops.
4. **Assess Results** Evaluate how students analyze the changes in the potential and kinetic energy of a car traveling on their roller coaster ride and how they use their analysis to present an argument that their design will work effectively (**NGSS MS-PS3-5, SEP7**). Check that students have correctly used the graphs of nonlinear functions to represent the structure of their roller coaster ride (**MP7**).

"Amusement Park and Roller Coaster Engineering." Ohio University.
https://onlinemasters.ohio.edu/amusement-park-and-roller-coaster-engineering/

"Roller Coaster." Chris Cavette, *How products are made*, 2018.
http://www.madehow.com/Volume-6/Roller-Coaster.html

Student Page Masters

APPLICATION TASK | **Create a Roller Coaster Ride**

Name: ______________________

Goal

Use nonlinear functions to model a roller coaster ride.

Language Objective

Using mathematical language and linking words and phrases to connect ideas, describe the use of nonlinear functions in modeling real-world situations.

Why Use Nonlinear Functions to Model Real-World Situations?

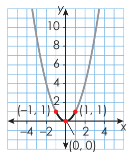
Sometimes the relationship between two real-world quantities is not linear.

Essential Question **How can you model real-world situations with nonlinear functions?**

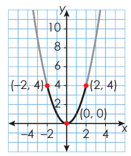
In this task, you are creating a **roller coaster** ride from graph sections of the **nonlinear** functions $y = x^2$ and $y = -x^2$. You will develop two plans for different roller coaster rides.

Constraints:

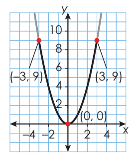
- The roller coaster ride can start at any height, but there must be at least two hills.
- Sections can be shifted up and to the right in whole-number increments.
- The heights of the hills should decrease from left to right.



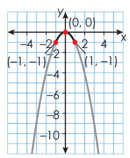
$y = x^2$: $-1 \le x \le 1$



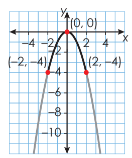
$y = x^2$: $-2 \le x \le 2$



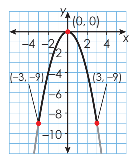
$y = x^2$: $-3 \le x \le 3$



$y = -x^2$: $-1 \le x \le 1$

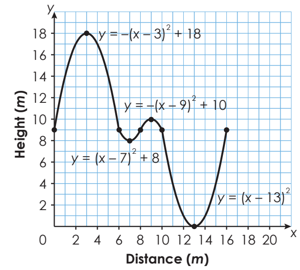


$y = -x^2$: $-2 \le x \le 2$



$y = -x^2$: $-3 \le x \le 3$

SAMPLE PLAN



Increasing Height Intervals	Decreasing Height Intervals
$0 < x < 3$ $7 < x < 9$ $0 < x < 3$	$3 < x < 7$ $9 < x < 13$

Did You Know? **The 570-foot-tall Skyscraper in Orlando, Florida, will be the world's tallest roller coaster.**

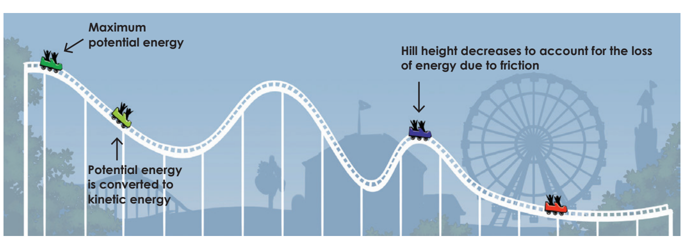
Background

Name: ______________________

A roller coaster car goes up and down **steep** hills as it travels across the coaster track. To begin with, the car is pulled to the top of the first hill by a mechanical **winch**. As it is pulled, the car acquires energy that subsequently decreases due to **friction** as the car rubs against the roller coaster track.

There are different kinds of energy involved in a roller coaster ride. **Potential energy** is the energy stored in the roller coaster car that has the potential to be used. Potential energy is often referred to as energy at rest. To illustrate this, look at the diagram shown. The roller coaster car has the maximum amount of potential energy when it is at the highest point, right before it is released to begin the ride.

Energy cannot be created or destroyed, but it can be converted into different forms for use. With this in mind, you will note that when the roller coaster car begins to move down the hill, the car's potential energy is being changed into **kinetic energy**, which is energy in motion.



Think about It How is this type of model similar to and different from a real roller coaster? Why might a model be convenient to use? Use linking words and phrases in your answer.

A real roller coaster and this model are similar because the overall shape and path that the roller coaster travels are represented. On the other hand, a real roller coaster is much larger and more detailed than what the model can show. Because the model shows a simplified version of the roller coaster ride, it can be easier to understand.

ACADEMIC VOCABULARY

Supporting Words

friction: the resistance of motion when one object rubs against another object, slowing each other down

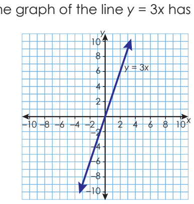
kinetic energy: the energy that is in motion

potential energy: the stored energy in an object based on how it is positioned

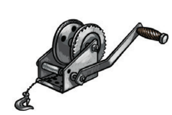
roller coaster: a ride at an amusement park with cars that roll on tracks with sharp curves and steep hills

steep: describes a sharp slope

Example: The graph of the line $y = 3x$ has a steep slope.



winch: a device used to lift or move an object, turned by a crank or other power source



Student Page Masters *(continued)*

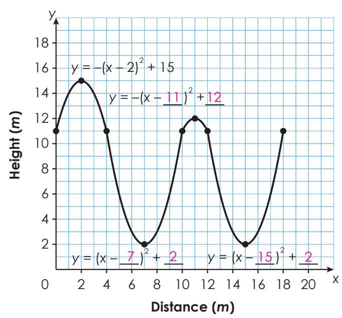
A | Understand

Name: ____________________

Solve a similar problem A **continuous** roller coaster ride starting at x = 0 has been created using 4 graph sections from page 1.

Analyze the graph to see how it meets each constraint. To begin with, complete the equations to label each section that was shifted. Then, complete the table to describe the height intervals for the roller coaster.

Constraints	Equation Notes
• There must be at least two hills. • The heights of the hills must decrease from left to right. • The roller coaster can start at any height.	$y = (x - h)^2 + k$ $y = -(x - h)^2 + k$ h: units shifted right k: units shifted up



Increasing Height Intervals
$0 < x < 2$
$7 < x < 11$
$15 < x < 18$

Decreasing Height Intervals
$2 < x < 7$
$11 < x < 15$

Think about It Use the vocabulary from page 4 to complete the sentence.
The roller coaster is made from nonlinear graph sections that are continuous.

Explain your reasoning using linking words and phrases and mathematical language.
Sample answer: Due to the fact that there are no gaps between the graph sections, the roller coaster ride is continuous. To illustrate, if we traced the graph, it could be traced with one single stroke.

Talk about It Talk about how to solve this problem with a partner. Discuss how the sample solution in ***section A*** meets the requirements of the task. Is there another way that you could solve the problem?

LINKING WORDS AND PHRASES

Mathematicians use linking words and phrases to link related ideas as they solve real-world problems. Below are some possible words and phrases you can use as you write and discuss creating a plan for a roller coaster.

due to the fact that
frequently
importantly
on the other hand
since
subsequently
to illustrate
with this in mind

Previously used linking words and phrases:

as a result
because
consequently
for instance
to begin with

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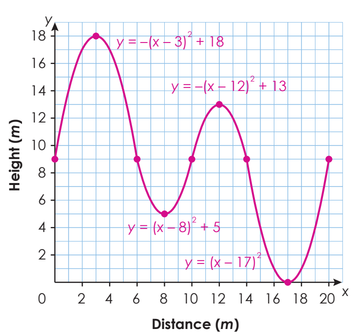
B | Organize

Name: ____________________

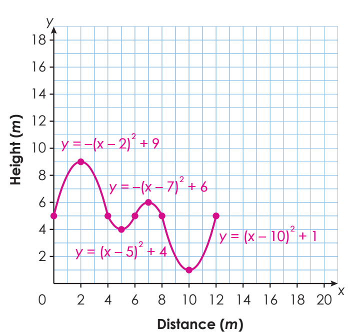
Create two continuous roller coaster rides starting at x = 0. Add labels to show the equation for each section after it is **translated**.

Constraints	Equation Notes
• There must be at least two hills. • The heights of the hills must decrease from left to right. • The roller coaster can start at any height. • Use the 4 graph sections from page 1 for the roller coaster ride. • Shift sections up and to the right in whole-number increments.	$y = (x - h)^2 + k$ $y = -(x - h)^2 + k$ h: units shifted right k: units shifted up

Plan 1



Plan 2



Explain It How is the **vertex** of each graph section related to where the height is **increasing** and **decreasing**? Use linking words and phrases to explain your reasoning.
Sample answer: It is important to realize that a vertex that is a minimum point is where the height changes from decreasing to increasing, and that a vertex that is a maximum point is where the height changes from increasing to decreasing.

MATHEMATICAL LANGUAGE

Major Words

continuous: going on without a gap or interruption

decreasing: Describes a section of a graph or function where the y-values decrease as the x-values increase. The graph section appears to be going down from left to right.

increasing: Describes a section of a graph or function where the y-values increase as the x-values increase. The graph section appears to be going up from left to right.

linear: Describes a function that has a constant rate of change. The graph of a linear equation is a straight line.

nonlinear: Describes a function that does not have a constant rate of change. The graph of a nonlinear equation is not a straight line.

translation: a transformation that moves a figure or object a certain distance without changing it in any other way

vertex: the minimum or maximum point of a parabola

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Student Page Masters *(continued)*

C | Solve

Name: ______________________

Complete the table for each of your plans. Fill in the blanks to describe where the height is increasing and decreasing over time for the roller coaster ride. Then analyze the plans.

Plan 1

Increasing Height Intervals	Decreasing Height Intervals
$0 < x < 3$ $8 < x < 12$ $17 < x < 20$	$3 < x < 8$ $12 < x < 17$

Plan 2

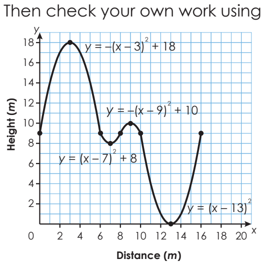
Increasing Height Intervals	Decreasing Height Intervals
$0 < x < 2$ $5 < x < 7$ $10 < x < 12$	$2 < x < 5$ $7 < x < 10$

Explain It Why are < signs used to describe where the height is increasing or decreasing rather than ≤ signs?

Sample answer: Since the height is neither increasing nor decreasing at the maximums and the minimums of the hills, those points are not included for the values of x.

D | Check

You can check your work from ***sections B*** and ***C*** by solving equations to find the vertex coordinates for each graph section. Complete the table for the Sample Plan. Then check your own work using the same process.



Equation	Vertex (*x*-value)	Solve for *y*
$y = -(x - 3)^2 + 18$	3	$y = -(3 - 3)^2 + 18 = 18$
$y = (x - 7)^2 + 8$	7	$y = (7 - 7)^2 + 8 = 8$
$y = -(x - 9)^2 + 10$	9	$y = -(9 - 9)^2 + 10 = 10$
$y = (x - 13)^2$	13	$y = (13 - 13)^2 = 0$

Explain It What should the result be when you solve each equation for *y* if the work is correct? Sample answer: The value of *y* should be the height of the roller coaster at the corresponding value of x for each vertex.

CONNECT TO SCIENCE

Roller coaster cars do not have engines. A mechanical winch pulls a roller coaster car up to the maximum height of the first hill.

What causes the car to move after it reaches the maximum height of the first hill?

Sample answer: The force of gravity causes the car to move downward. The car has stored energy as it reaches the top of the first hill. As a result, it can go up and down other hills on the way down.

Why must the heights of the hills decrease as the ride continues?

Sample answer: Since the car loses energy as it rides along the tracks, it consequently will not be able to climb a hill that is higher than the one it just descended.

Extend Create a longer roller coaster ride starting at x = 0 to a maximum distance of x = 40. In addition to showing a greater length of the x-axis, what other changes will you need to make to your display of the graph? Explain.

Sample answer: Because the hills must decrease in size from left to right, the roller coaster will have to start at a greater height. The display will also have to show greater values along the y-axis.